Subsidizing Renewables in the Presence of a Dirty Backstop

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Abstract

As the world's demand for fossil fuels grows stronger every year it is important to understand how to balance the consumption of fossil fuels with the damage to the environment. Optimal climate policy is studied in a calibrated model that includes not only an exhaustible resource with stock-dependent price of fuel, but also an infinitely elastic supply of both a clean expensive and a cheap dirty backstop. We focus on the case where a first best tax on carbon is unavailable, and rely on a second best strategy. Unlike many other theoretical studies that use integrated assessment models we calibrate directly to real-world values. As a result of this calibration we find it is optimal to subsidize renewables only after coal is introduced. Furthermore, we find that even the optimal subsidy performs comparatively poor at combating climate change compared to a first best tax. The subsidy allows excessive carbon emissions from dirty coal leading to significant damages due to climate change. We conclude that it is important to be wary of using this second best tool as a policy instrument.

Keywords: renewable resources, dirty backstop, subsidizing renewable energy, climate change, Green Paradox, resource extraction, optimal control

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1. Introduction

Climate change is regarded as one of the most challenging issues our world has to face. In order to avoid some of the negative consequences of climate change (Intergovernmental Panel on Climate Change, 2007; Stern, 2007) substantial climate policy is needed to curb to emissions of CO_2 from not just scarce fossil fuels like oil and gas[3], but also from environmentally worse and abundant coal (Edenhofer, Knopf, Kalkuhl, Fischer, Treber, Bals, and Luhmann, 2009). Currently, oil is still a competitive alternative to coal. However, looking at the world's rising energy diet (World Bank, World Development Indicators, 2012), it is inevitable that rising extraction costs will make oil uncompetitive relative to coal. Unless suitable environmental policies are put in place to curb the use of fossil fuels and replace them with renewables, this will be disastrous for the earth's climate (Sinn, 2008). A substantial carbon tax or permit system is needed to save the climate (van der Ploeg and Withagen, 2010), but may in some cases be politically infeasible. It is therefore important to examine the impact of various other climate policies with an availability of both coal and oil. This study contributes to that goal by finding optimal second best policy in the presence of a dirty backstop and comparing it to the market outcome and first best tax. We also examine in closer detail the effects of different damage and cost specifications, as well as announcement effects. Our sad conclusion is that even the optimal subsidy hardly increases welfare and that for substantial welfare improvement governments need to turn to carbon taxes or tradeable permits.

We study a theoretical framework where utility is derived from energy input, and we assume three kinds of energy input (oil, coal and renewables) as perfect substitutes[4]. We assume oil is exhaustible and extraction costs are stock-dependent, but coal and renewables are assumed abundant. Initial extraction

[3] From here on out, we shall refer to oil and gas as 'oil'

[4] This assumption is made for technical convenience. For a related study where this is not the case, see Smulders and van der Werf (2007)

cost of oil is assumed to be lower than that of coal, but this changes over time as the stock of oil becomes smaller. Social welfare is found by subtracting climate damages from utility, discounting and integrating over time. Climate damages are cumulative in the emission strength of each fuel type, where any natural degradation is abstracted from for technical ease. In the long run, coal emits more CO_2 than oil, but is relatively cheap; renewables are expensive and carbon-free. In practice, there are many more severe negative externalities from burning coal than just the exacerbation of global warming, like air pollution and a public health burden, but we do not consider those here (Epstein, Buonocore, Eckerle, Hendryx, Stout III, Heinberg, Clapp, May, Reinhart, Ahern, Doshi, and Glustrom, 2011). It should further be noted we do not include any technological change. This can have two effects: development of extraction technologies might lead to faster depletion and more emissions, whilst on the other hand improvement of renewable energy efficiency might lead to lower prices of renewables and thus speed up the introduction of said renewables.(Acemoglu, Aghion, Bursztyn, and Hemous, 2012; Fischer and Newell, 2004).

Our work follows a long line of literature in calculating optimal policy for resource extraction with a climate change externality. Models of optimal resource extraction policy have been studied for a long time (Hotelling, 1931; Dasgupta and Heal, 1974). More recently, many studies have considered optimal resource extraction and carbon taxation with climate change and a renewable backstop (Hoel, 2011; van der Ploeg and Withagen, 2010). The models in these studies allow for capital accumulation, but do not include the presence of a cheap dirty backstop. Others do include a dirty backstop, like Smulders and van der Werf (2007), who show how restricting CO_2 emissions affects resource prices and extraction over time in a partial equilibrium model with imperfect substitution of energy inputs. Chakravorty et al (2008; 1997) also consider both a dirty and a clean backstop. They extend the Hotelling theory to resources differentiated by their pollution characteristics to examine the sequence of optimal extraction in the presence of a carbon emission ceiling, where they find cleaner fuels are used up earlier. A study that comes particularly close to ours is the one by van der Ploeg and Withagen (2011). They also study optimal climate policy in a setting with a cheap and dirty backstop in a similar model. Although they analytically derive closed-form solutions for the extraction paths where we only find ours numerically, they do not consider the optimal second best policy. We calibrate our parameters to real-world values, and find different regimes than Ploeg and Withagen (2011) found for their calibration of the model.

Our main contribution is finding the optimal level and timing of a subsidy on renewables when a carbon tax is politically or otherwise infeasible. We show that, given our calibration of the model, it is optimal to subsidize renewables to just below the price of coal, well after the market has phased out oil, but much earlier than renewables would be introduced in the first-best outcome. We also demonstrate, that given a high cost of renewables, this optimal subsidy is only marginally welfare improving compared to a tax. Furthermore, we examine the case when the social planner has to announce the date of implementation of the subsidy at the very beginning. Previous studies have raised concerns that such an announcement effect may lead to a green paradox (Sinn, 2008; van der Ploeg and Withagen, 2012a; Grafton, Kompas, and Long, 2010): a subsidy on carbon-free renewables exacerbating global warming damages by incentivizing faster depletion of fossil fuels. We show that, contrary to the aforementioned studies, there is no green paradox when renewables are subsidized after the market phases in coal. Hence, we find that as long as the pre-announced subsidy comes after the market phases in coal, the early announcement has no effect on the fossil fuel extraction.

Lastly we examine the sensitivity of our results to our calibration. We study how the optimal timing and level of subsidy changes with varying sensitivity of environmental damages to carbon emissions (a key parameter about which there is much controversy). We find that our results are fairly robust to these variations. The damage parameter has to be unrealistically high for it to be optimal to introduce a subsidy on renewables as soon as the market phases in coal. Only for extraordinary damages (twice the number that is suggested by Nordhaus (Nordhaus, 2008)), it is optimal to subsidize renewables below the price of coal, to induce an earlier phasing out of oil.

The rest of the paper is organized as follows. In section 2 we describe the model. In section 3 we describe the characteristics of the social optimum and show the optimal extraction paths and the calibration of our model, whereas in section 4 we study the optimal level and introduction time of the subsidy. Section 5 concludes. Proofs and numerical methods are found in the appendix.

2. Model

We adopt the framework presented in van der Ploeg and Withagen (2011). We study both optimal resource extraction and second best policy in a setting where coal and renewables are phased in instead of oil or alongside oil. Fuel types are treated as perfect substitutes and the supply of both coal and renewables is infinitely elastic. Emissions from burning oil and coal increase the CO_2 concentration in the atmosphere. As natural decay is extremely slow(Archer, Eby, Brovkin, Ridgwell, Cao, Mikolajewicz, Caldeira, Matsumoto, Munhoven, Montenegro, et al., 2009) and makes the problem much harder, it is abstracted from, and the CO_2 stock evolves according to:

$$\dot{E} = q(t) + \psi x(t), E(0) = E_0, \psi > 1, \tag{2.1}$$

where E, q and x denote atmospheric CO_2 concentration, oil use and coal use, respectively. The emission coefficient of oil is normalized to one, and the emission coefficient of the dirtier coal is thus bigger than one. Utility is gained solely through the usage of energy, and the utility function is assumed to be concave and increasing. Further, global warming damages are the only externality considered and they are dependent on accumulated emissions of CO_2 only. The cost of energy usage is subtracted from utility, and we assume that the cost of both the renewable backstop and the abundant coal stocks are constant. This way, the social planner problem reads:

$$\max_{x(t),q(t),z(t)} \int_0^\infty e^{-\rho t}[U(q(t)+x(t)+z(t))-G(S(t))q(t)-bx(t)-cz(t)-D(E(t))]dt \tag{2.2}$$

subject to equation 2.1, $x(t) \geq 0$ and the oil depletion equation:

$$\dot{S} = -q(t), S(0) = S_0, q(t) \geq 0, S(t) \geq 0 \tag{2.3}$$

Here ρ is the rate of time preference, G is the per unit of extraction cost of oil, b and c the per unit cost of coal and renewables, respectively, and z(t) is the use of renewable energy. Note that equation 2.3 implies that $\int_0^\infty q(t)dt \leq S_0$. The problem of the market is similar to the social planner problem, but acts as though $D = 0$.

The policymaker achieves the optimal (first-best) allocation by levying a tax τ on carbon. If a carbon tax is not available, the policymaker uses a second best policy of subsidizing renewables. The policymaker then levies a subsidy of $\eta(t)$ replacing c by $c - \eta(t)$ in the market welfare expression. The policymaker then optimizes with respect to $\eta(t)$, to find the level that maximizes social welfare.

3. Social Optimum

3.1. Solving the Social Optimum Problem

In this section we look for an optimal allocation of energy consumption over time, a solution to 2.2. We also examine the solution to the laissez-faire problem, and calculate the optimal level of carbon tax needed for the market to arrive at the optimal allocation. A special case of our social optimum with renewables is the optimal case without renewables described in van der Ploeg and Withagen, when $c \to \infty$.

Below are the first order conditions for the social planner problem 2.2 (optimal policy with oil, coal and renewables as options), where λ and μ are the costate variables:

$$U'(q+x+z) - G(S) \leq \lambda + \mu,\ q \geq 0, c.s., \quad U'(q+x+z) - b \leq \phi\mu,\ x \geq 0, c.s. \quad (3.1)$$

$$\dot{\lambda} = \rho\lambda + G'(S)q, \qquad \dot{\mu} = \rho\mu - D'(E), \quad (3.2)$$

$$U'(q+x+z) - c \leq 0,\ z \geq 0, c.s., \qquad \lim_{t\to\infty}[\lambda(t)S(t) - \mu(t)E(t)] \quad (3.3)$$

The interpretation of the first of inequalities 3.1 is that if the marginal value of using oil is less than the social value of keeping oil in the ground including the value resulting from less global warming damages, no oil is used. If oil is used, the first order condition holds with equality. The second expression in 3.1 states that coal is not used if the marginal value of coal is less than its marginal global warming cost. If coal is used, its marginal utility must equal the marginal cost of extraction including the damage done by global warming. The rest of the first order conditions give the dynamics of the shadow value of oil and the social cost of CO2, and the transversality condition.

Note that not all first order conditions have to hold with equality at every point in time, but instead the optimal allocation is separated into a sequence of regimes. Each regime is defined by which resources are being utilized in the economy (oil only, coal only, both oil and coal or renewables only) and are characterized by a separate set of first order conditions. The optimal sequence of the regimes depends on the assumptions we make about the functional forms, such as the extraction costs of oil, and the calibration of our functions. In this paper we make the following assumptions:

Assumption 1.

1. *As in van der Ploeg and Withagen (2012b), we assume that c is such that renewables are only introduced after coal is phased in*
2. $b + \psi D'(E_0)/\rho > G(S_0) + D'(E_0)/\rho$, *(no simultaneous use of oil and coal at start),*
3. $\exists 0 < S_1 < S_0$ *such that* $b + \psi D'(E_0 + S_0 - S_1)/\rho = G(S_1) + D'(E_0 + S_0 - S_1)/\rho$ *(In the social optimum, as the oil stock depletes, at some point the social cost of oil and coal is the same and it becomes optimal to use oil and coal simultaneously)*
4. $U'(0) > b + \psi D'(E_0 + S_0 + \hat{Y})/\rho$, *where* $\hat{Y}$ *is defined by* $G(0) = b + (\psi - 1)\frac{D'(E_0+S_0+\hat{Y})}{\rho}$ *(marginal utility of energy is high enough to warrant production of oil or coal).*

The intuition behind the first assumption is that in the first period the marginal cost of coal is greater than the marginal social cost of oil, thus ensuring that there is no simultaneous use in the beginning. Similarly, the second assumption ensures that at some level of oil stock, marginal social costs of oil and coal are equal ensuring some simultaneous use. The last assumption ensures that in the beginning the marginal benefit of some oil and coal is higher than the marginal cost - ensuring positive energy use. The regime sequence these assumptions lead to is outlined in the proposition below. The main conclusion is that there only exists a coal-only phase when there is oil left in situ.

Proposition 1. *In the social optimum, with sufficiently high cost of renewables, there is a sequence of three or four regimes: either oil-only, oil-coal, and renewables, or oil-only, oil-coal, coal-only and renewables-only. The case where there are only three regimes occurs when*

$$U'(x(\infty)) = G(\bar{S}) + \mu(\infty) = G(\bar{S}) + (U'(x(t = \infty)) - b)/\psi \tag{3.4}$$

has a positive solution for $\bar{S}$, the final level of oil.

Proof. See van der Ploeg and Withagen (2011). □

Note that equation 3.4 has a positive solution for $\bar{S}$ when $G(S)$ is proportional to the inverse of S. In that case, the last drop of oil comes at infinite cost. If the function is taken to be linear, there is no guarantee for such a solution. The regime changes thus differ with the shape of $G(S)$.We define T_1 as the time where simultaneous use begins. It follows directly from the optimality equations that:

$$b - \psi \frac{D'(E(T_1))}{\rho} = G(S(T_1)) - \frac{D'(E(T_1))}{\rho} \tag{3.5}$$

when simultaneous use occurs. The interpretation of this equation is that the discounted marginal cost of energy minus the discounted marginal damages should be equal for coal and oil, respectively. The social optimum will start using oil, as follows directly from our assumptions. Then, when 3.5 holds for the first time, a switch will occur from using oil-only to using both oil and coal.

Let T_2 be the time when the simultaneous use regime ends. As outlined in Proposition 1 there are two possibilities after T_2. In the first case all oil is exhausted ($S(T_2) = 0$) and the coal-only phase begins. The optimal consumption of coal is then defined by the appropriate first-order condition. In the second case some oil is still left in situ and the economy switches to renewables. Which case occurs depends on whether condition 3.4 holds or not. We assume that the condition holds, and some oil is left in situ after the end of simultaneous use. While it is not right away clear that this expression must hold, we will demonstrate that in our calibration it does indeed. Therefore the switch time T_2 at the end of the oil-coal phase is also the time when renewables are introduced. Renewables take over when the cost of coal plus the shadow price of carbon is equal to the cost of renewables (or equivalently in the simultaneous use regime: the cost of oil plus the the carbon tax):

$$b + \psi\tau(T_2) = c, \text{ where } \tau(t) = D'(E(t))/\rho$$

After the introduction of carbon-free renewables, the economy continues without adding damages to the environment. The following proposition characterizes the renewables regime:

Proposition 2. *Suppose T_2 is the switch time from an oil-coal phase to the renewables-only regime. Then $D(E(t)) = \frac{\rho(c-b)}{\psi} \forall t > T_2$. Furthermore the energy consumption after the switch to renewables is given by $z(t) = (U')^{-1}(c)$.*

Proof. This follows directly from the first-order conditions. Addressing the second statement first, $z(t) = (U')^{-1}(c)$ follows from $U'(z) - c = 0$. The other statement follows from energy continuity: $U'(x(T_2^-)+q(T_2^-)) = b+\psi D'(E)/\rho = U'(z(T_2^+)) = c$. □

With this last proposition we have concluded the overview of the regimes in the social optimum. These will be used to simulate the outcomes given our calibration of this model.

3.2. Solving the Laissez-Faire Equilibrium

Compared to the optimal allocation the problem of the free market is slightly simpler. As opposed to four possible regimes in the first-best there are only two regimes in the free-market.

Proposition 3. *In the laissez-faire economy that acts like $D = 0$, there is no simultaneous use, moreover the different regimes are oil-only and then coal.*

Proof. See Appendix A.4 for the fact that there is no simultaneous use. Furthermore, when $D(E) = 0$ assumption 1 tells us that at start there will be an oil-only economy until oil is more expensive than coal, i.e. until 3.5 holds, now reduced to $G(S(T)) = b$. As there is no coal-oil regime and renewables at cost $c > b$ are expensive, there will be a switch to a coal-only regime, which continues indefinitely. □

As in our calibration $c > b$, renewables will not be phased in the market economy unless a subsidy is set at at least $c - b$, cf. section 4. Because of the continuity of energy use we have that $x(T) = q(T)$, and therefore also $U'(x(T)) = U'(q(T))$. Further, from the first order conditions it follows in coal-only regime that $U'(x(T)) = b$ and thus the final use of oil depends on the price of coal. These conditions are sufficient to characterize the laissez-faire regime.

3.3. Calibration

We used numerical methods to solve for socially optimal and free-market allocations. Hence, before presenting our results we must specify the functional forms and parameter values used. To calibrate our model, we use the following functional forms:

Assumption 2. *$G(S) = \gamma_1(\frac{S_0}{S})^{\gamma_2}$, $D(E) = \kappa E^2$, and $U(y) = \alpha y - \frac{1}{2}\beta y^2$.*

We calibrate our functional forms so that the values of variables correspond roughly to real-world values, expressed in ppmv and Trillion USD.

Damages

According to Nordhaus (2008) climate change damages at the current point are equal to .2% of GDP. Current global GDP is \$ 70 trillion, so current damages from climate are \$ 140 billion ($D(E_0) = .14$). Initial level of CO_2 concentration $E_0 = 388ppmv$. Calibrating $D(E_0) = \frac{\kappa}{2}E^2 = 0.14$ leads to $\kappa = .00000186$

Energy: Cost and Emissions

We follow Rezai, van der Ploeg, and Withagen (2012) on calibrating the oil extraction cost $G(S)$. Initial costs are calibrated at \$ 5 per barrel of oil. A barrel of oil is equivalent to 1/10 ton of carbon, and 2.13 GtC is equal to 1 part per million. We express all prices in \$trillion per ppmv, thus $\gamma_1 = 0.1$. According to IEA 2008, the extraction cost of oil will quadruple after 500 ppmv more will be extracted. We assume $S_0 = 2000$ ppmv so $\gamma_2 = 4.81$. From the IEA 2012 statistics report on CO_2 emission from fuel combustions (IEA, 2011) (see table 1), we get that the relative emission factor for coal lies between 1.55 and 1.37. We set the relative emission factor in between those two values, $\psi = 1.4$.

Further, we find from the IEA website that the real fossil fuel production prices are IEA (2011) 1.62 dollars per $10^6 Btu$. This translates to 0.00153 dollars per MegaJoule, or 0.0055 dollars per kWh. We express prices in $ per ppmv. However from an energy standpoint, coal is a perfect substitute for oil. Thus when calculating the price of coal we assume it has the same amount of grams CO_2 per kWh as oil does (670). Assuming 2.13 Gigatonnes of carbon is equivalent to one part per million by volume in the atmosphere (Rezai, van der Ploeg, and Withagen, 2012), we get $b = 0.17$ trillion dollars per ppmv.

The true renewable energy production price is difficult to estimate (as it will likely decrease over time). We take initial price of renewables to be around thrice the extraction cost of coal ($c = .51$), which agrees with the findings of the Renewables Global Status Report 2012 and the Nuclear Energy Institute (Nuclear Energy Institute, 2011; Rewables 2012 Global Status Report, REN 21, 2012).

Utility

The calibration of the utility function is fairly arbitrary, especially in a partial equilibrium model where utility is derived directly from fossil-fuel consumption. To calibrate the utility function we assume that current fossil fuel consumption corresponds to the market outcome. Therefore for the market solution initial emissions due to fossil fuels q_0 must be equal to 4.03 ppmv. Normalizing $\alpha = 1$ we get $\beta = .21$.

3.4. Policy Simulations

With our assumptions and calibrations we can solve for the social optimal and market outcomes and using numerical methods, displayed in figure A.1. A detailed description of how we arrive at our results using numerics is found in section Appendix A.2. The timing of the regimes in our results is as follows. The oil-only phase ends at $T_1 = 106.59$ in the social optimum compared to $T = 52.47$ in the market economy. Renewables take over at $T_2 = 262.37$ in the social optimum. We see that in social optimum oil use decreases over time in the first phase, where coal is not used yet. This decline in oil use is caused by increasing marginal damages of oil due to increasing CO_2 concentration, and due to an increasing cost of oil extraction. Once the shadow price of oil is equal to that of coal, simultaneous use begins. In this regime, the relative cheapness of coal outweighs its bad environmental properties, as coal is the predominant source of energy. Still the increasing environmental damages lead to a slow decline in both coal and oil usage during this stage. As environmental damages increase, the shadow price of coal and oil eventually becomes equal to the price of renewables, and they are phased in. As the energy demand remains the same, the amount of renewables used in the last regime is equal to the sum of oil and coal used in the earlier regime. Emission stock is growing convexly until the introduction of renewables.

Note that after renewables are introduced, there is still some oil left in situ with and the final oil stock $S = 1629.5$. This is one of the key differences between our result and that of (van der Ploeg and Withagen, 2012b), that relies on different assumptions in the functional forms. In their paper van der Ploeg and Withagen observe that in the social optimum all oil is exhausted, and there is a brief coal-only regime before renewables are phased in. The difference between our results is driven somewhat counter intuitively, by a steeper rise in oil extraction cost in our calibration. As oil becomes more and more expensive it is slowly substituted by coal but is never completely exhausted. We believe that this result is closer to the real world as we have used real-world values to calibrate the oil extraction function.

In contrast with the social-optimum in the market outcome, the economy relies solely on oil or coal depending on which is cheapest, and more fossil fuel is burned than in the social optimum at any given time. This result (in full agreement with the literature on climate change economics) is due to the fact that the market ignores the climate change externality. Thus the higher fossil fuel consumption leads to higher emissions, especially if the market switches to coal, where in the social optimum it should . Note further that in the social optimum more oil is depleted than in the market outcome. This occurs because oil is cleaner than coal. Thus even when the oil extraction cost rises above coal, the social planner keeps using oil to mitigate climate change, while the market does not. Welfare comparisons can be made, and in the market economy social welfare is 73.47 compared to 96.70 in the social optimum. Thus we can conclude that the utility from higher fossil fuel consumption is significantly outweighed by higher environmental damages, resulting in 25% welfare loss.

4. Second Best Policies

The first-best situation may be unattainable because a carbon tax is often politically or otherwise unfeasible. We therefore turn to finding the optimal second best policy where a social planner can levy a subsidy on carbon-free renewables. We assume that the social planner instrument is to announce the subsidy at any time and implement it immediately upon announcement. Mathematically, the outcome for the second best economy is comparable to that of the market outcome, except that the normally constant cost of renewables c is now replaced by the time-varying cost $c-\eta(t)$, where $\eta(t)$ is the level of subsidy given on carbon-free renewable energy resources chosen such as to maximize social welfare. Because we treat energy inputs as perfect substitutes, the market will choose the cheapest form of energy consumption. Therefore, a subsidy will only change the energy usage when it is higher than $c-b$ or $c-G(S)$, depending on the question if oil or coal is the cheapest fuel type. As there is no externality on carbon-free renewable energy usage, there is no reason to use any level of subsidy higher than this level. This leads to the conclusion that the best level of subsidy is constant, and given by $b-c+\epsilon, \epsilon \to 0$, if $b < G(S(T^*))$ or $G(S(T^*)) - c + \epsilon, \epsilon \to 0$, where T^* is the time of subsidy introduction. The optimal subsidy is then solely determined by the timing of the subsidy T^*. This is of course will not hold if the social planner cannot control the timing of the subsidy. We examine that case later in Section 4.2

All that remains to do is to find the welfare maximizing timing of the subsidy. Before we do that we must understand how the timing of the subsidy affects the total discounted welfare. The effect of the subsidy on welfare can be broken down into two parts. On one hand the subsidy is beneficial as it stops all future fossil fuel use, reducing future environmental damages. On the other hand, the subsidy itself costs the society (we assume that the government finances the subsidy through a lump sum tax, passing the cost to the society without distorting the outcome). The question is which of the two effects is dominant, and at what time, the difference between the positive and the negative welfare effect of the subsidy is the largest. Recall that we assumed that the social planner can instantly announce and implement the subsidy at any time. Therefore up until the timing of the introduction of the subsidy the economic outcome is the same as that of the market. After the subsidy is introduced, renewables are phased in with demand for renewables equal to market demand for energy at the subsidy introduction time. Using the market solution we can then compute the social welfare for every possible subsidy timing T and find T^*, the timing which maximizes total discounted welfare.

4.1. Policy Simulations

As explained earlier, given that the subsidy is effective immediately upon announcement, the timing of the subsidy determines the level of the subsidy. In figure A.3 we present a plot of total discounted welfare as a function of the timing of subsidy. The following is the intuition behind the subsidy's timing effect on welfare. During the oil-only stage the low price of oil makes the subsidy expensive while the environmental damages are too small. Thus delaying the subsidy until after the market phases in coal increases welfare. Even after coal is introduced, environmental damages are still too low to justify the subsidy. Thus the welfare-maximizing subsidy timing is at $T = 164.02$ - significantly after the market introduction of coal (52.47). Furthermore, this switchf time is much earlier than the switch time $T = 262.37$ in the first-best social optimum. Recall that here we assume that the subsidy is a surprise, announced only at the time when it is implemented(we will examime announcement effects in section 4.2). Hence the policy maker does not affect the behavior of the market before the subsidy is introduced. The result, as can be seen in figure A.2, is that before the subsidy is introduced the level of oil, and more importantly coal consumption is equal to the market outcome - significantly higher than the first best level. To compensate for that excess environmental damage the policymaker uses the subsidy to phase in the renewables at a significantly earlier time than the first best. As before the introduction of a subsidy on renewables the economy is identical to the market economy, there is a larger amount of oil left in situ($S = 1791$).

The welfare gained from the subsidy is fairly small: 76.56 compared to 73.33 in the lassiez-faire case, and 96.70 in the social optimum. We thus conclude that given our calibration, the subsidy is a fairly inefficient policy. The high price of renewables is what drives this result. In our calibration, the cost of renewables is triple that of coal, making the subsidy very expensive. As a result the subsidy is only welfare improving for fairly high levels of CO_2 concentration. Before that concentration is reached a significant amount of damage occurs, significantly reducing welfare.

4.2. Subsidy Announcement Effect

In the previous section we found the optimal timing for a subsidy, given that the social planner can announce the subsidy at the time of its implementation. In other words, we assumed that the implementation of the subsidy can come as a surprise to the market. However, that is not always possible. Once the government plans to implement a policy it is usually known well in advance (simply by the virtue of how many people are involved in the implementation of the subsidy). For this reason we explore the case when a subsidy can be implemented at time $t = T_1$ but has to be announced at time $t = 0$.

Concerns have been raised in regards to such a subsidy scheme (Sinn, 2008).

An announced subsidy on renewables will decrease the value of the fossil fuel stock after implementation at $t = T1$ giving incentives for the market to extract fossil fuel faster before the subsidy is introduced. This phenomenon called the "green paradox" makes subsidies potentially counterproductive. There are three types of green paradoxes possible:

- Frontloading of oil extraction with no negative effects on welfare (relative to market)
- A decrease of "green welfare" (increased environmental damages due to emissions)
- A decrease in total welfare

However, no type of green paradox will occur if the subsidy is announced to be introduced after the market phases in coal. This is formalized in the following proposition.

Proposition 4. *There is a green paradox if and only if the switch to renewables happens before coal is introduced.*

Proof. See section Appendix A.3. □

As we have seen in the previous section, given our calibration it is only optimal to introduce the subsidy after the market phases in coal. Thus in our model, no green paradox occurs. The early announcement of the subsidy does not affect the market behavior in any way, there is no welfare loss, green welfare loss or excessive oil extraction.

These results are of course conditional on our calibration. With significantly higher climate damages or lower cost of renewables, it may be optimal to introduce the subsidy before the market phases in coal. In that case, announcement effects start to play a role. However, as we will demonstrate in section 4.3 the parameter values must change significantly in order for that to happen, making the "no Green Paradox" result fairly robust to a large range of calibrations.

4.3. Varying Damage Estimations and cost of renewables

Amongst politicians, economists and in the scientific community there is much disagreement about the size of damages caused by global warming. We have used the damage calibration of (Nordhaus, 2008) but there are other studies (Tol, 2002) that report much higher values of climate damages. It is therefore important to check how sensitive our results are to a given calibration of climate damages. We thus set out to investigate the effect on our results of changing the amount of damages resulting from one unit of emissions. Remembering $D(E) = \kappa E^2$ in our model, we look what the effect is of changing κ in a second best setting. We plot how the optimal switch to renewables depends on the value of κ in figure A.4. Here there is no announcement effect: the subsidy is announced at the time it is introduced. This means the market doesn't know the subsidy is coming.

What we see is that for relatively small values of κ a subsidy is put in place only after coal is introduced: the marginal cost of emitting coal is initially still outweighed by the utility gain from using the cheaper coal instead of the expensive renewables. In the limit $\kappa \to 0$ we see that the optimal time of introducing subsidies approaches infinity, which is intuitive as we expect no subsidy is necessary when there is no environmental externality. For relatively large values of κ we see that a subsidy is introduced at time zero: the damages are so large that even for the initial stock of carbon in the atmosphere, it is suboptimal to use coal. Note that for this to happen the damage specification must be seven times as large as in Nordhaus (2008). For values of κ in between these extremes, we see that either renewables are already introduced right after the oil-coal regime, before coal-only is introduced. As κ, gets larger the switch happens sooner. Or otherwise, renewables are introduced after the oil-coal regime but before a switch is made to a coal-only economy. As κvaries the optimal switch time now remains the same. This regime exists because the marginal damages from burning coal are more severe than of burning oil, and it thus requires precautionary measures to avoid the severe damages from switching to a coal-only economy. As can be seen in the figure A.4 in order to justify subsidizing renewables before the switch to coal κ must be equal to 710^{-6} meaning that damages must be three times larger than what (Nordhaus, 2008) estimates them to be. Even with much uncertainty about climate damages such large damages are unlikely, and we can thus tentatively conclude that it would never be optimal to subsidize renewables before the economy switches to coal. To do some pure welfare analysis, look at figure A.6, where welfare is plotted for both the market outcome and the subsidy case for various levels of κ. We see that the welfare gains from a subsidy are larger if the damages of global warming are larger, and the rate at which the effect becomes larger is also larger as damages grow.

Another parameter we vary, as it is difficult to calibrate precisely, is the cost of renewables. We plot in figure A.5 the optimal time to introduce a subsidy and hence switch to renewables for a range of values for c. We see that if renewables are really cheap it is optimal to start subsidizing right away, and as they become more expensive this switch time increases. This makes sense as for a higher cost of renewables a higher subsidy is need to make them competitive. There is a region where the switch time remains the same even though the cost of renewables increases, at exactly the switch time from the oil-coal regime to the regime of coal-only. This can be interpreted as that it is welfare-maximizing to never enter the coal-only phase. In order to avoid the negative externality of relying on coal completely, a switch will be introduced even given the cost of renewables. The subsidy then has the same effect as a prohibitive tax on coal. As can be seen in the figure, only for c below .26 is it welfare-maximizing to subsidize renewables at the price of oil, which means must be only 1.5 times the price of coal to change our conclusions. This is towards the very bottom of the range of renewable energy costs (according to the IEA). Hence, it is highly likely that renewables are still too expensive for a subsidy to be an effective environmental policy. Welfare gains from a subsidy are shown in figure A.7 by plotting both the market welfare and the subsidy-case welfare at the optimal switch time for different values of the cost of renewables c. We see that the welfare gains are asymptotically zero as the price of renewables c goes up, and welfare gains from a subsidy are increasing with cheaper renewables. It becomes more profitable more rapidly as the price of renewables c goes down. We estimated that currently we are at $c = .51$, which, looking at the graph, implies a lot of welfare gain can be achieved by making renewables cheaper. This confirms

Lastly, we examine which values of c and κ violate assumption 1 which ensure a certain regime sequence. From these assumptions the first ensures that renewables are introduced only after coal. That requires that the optimal starting point for simultaneous use is earlier than the optimal starting point for renewables use. In other words $T_1 < T_2$ (or $E(T_1) < E(T_2)$ since there is no natural CO_2 decay) where T_1 is defined by

$$b + \psi D(E(T_1))/\rho = G(S_0 - (E(T_1) - E_0)) + D(E(T_1))/\rho$$

and T_2 is defined by

$$c = G(S_0 - (E(T_2) - E_0)) + D(E(T_2))/\rho$$

For a range of values of κ and c we find $E(T_1)$ and $E(T_2)$. Holding renewables cost at our initial value $c = .51$ we find that for $\kappa > 3.2\ 10^{-6}$ $E(T_2) < E(T_1)$, hence coal is never introduced. Similarly holding $\kappa = 2\ 10^{-6}$ we find that coal is never introduced for $c < .42$. Note that these damage parameter values are significantly higher (and renewables cost significantly lower) than those for which the green paradox occurs. Hence we can confidently say that as long as the damage function adn cost of renewables does not violate our assumptions for the optimal regimes, no green paradox occurs for the optimal second best policy.

5. Conclusion

In this paper, we study the optimal environmental policy with a clean and dirty backstop in a theoretical model of resource extraction including environmental damages from CO_2 emissions. The importance of including a dirty backstop lies in the real-world abundance of coal, and the impact its presence has on oil extraction. In addition to finding the first-best policy we focus our attention on a case when a carbon tax is not politically feasible and the social planner can levy a renewable energy subsidy instead.

Assuming oil is cheap at the start and renewables are expensive, we find that depending on the calibration of the model the sequence of energy inputs used is either oil, oil-coal, renewables or oil, coal, renewables. After calibrating our model to real-world values we find that when optimal environmental policy is pursued the economy never switches to only coal, simultaneously burning oil and coal until renewables are phased in. This means the ordering does not depend on just the cost of fuels, but on its emission coefficient as well, clearly breaking with the Herfindahl rule. Compared to the laissez faire outcome, renewables are introduced in the social-optimum once the environmental damages outweigh the benefits of the lower price of renewables.

We find that in the optimal second best case the sequence of resource regimes is oil, coal, renewables, and no green paradox occurs when a dirty backstop is present. The optimal renewable subsidy results in renewables being phased in after oil is phased out but significantly earlier than in the first-best case. A subsidy results in a green welfare improvement over the market outcome, but between the optimal subsidy and the first-best policy there a substantial welfare gap due to excessive coal consumption before renewables are phased in. We thus conclude that a subsidy on renewable energy is hardly welfare improving relative to the carbon tax.

The main policy-oriented result of this paper is that second best policies such as subsidies are highly inferior to the first-best carbon tax when a dirty backstop is availiable. The high cost of renewables makes subsidies expensive, while cheap and dirty coal leads to excessive polution when unchecked by a carbon tax. This paper is thus a warning for an important mechanism: when renewables are subsidized, the market for fossil fuels is distorted, so that there positive welfare effect will be small. Thus we conclude that a policy maker should push for a carbon tax by any means possible and ignore the politically convenient but highly ineffective renewable subsidy.

As there is no clear agreement on the value of some parameters in our model, we discussed how our model behaves under different values for them. Specifically, the damage parameter and the cost of renewables were discussed. We find that our conclusions are somewhat sensitive to these parameter values: for relatively cheap renewables and high climate damages it becomes optimal to subsidize renewables at the price of oil to avoid any coal use in the economy. However, the range of these parameter values is far from our best estimates today.

Our model is stylized, abstracting from many real world processes such as capital accumulation, technological change, learning by doing, or natural decay of CO_2. One extension is to see how these features affect our results. Specifically, it is unclear whether the absence of a green paradox would still hold if the price of renewables could be changed through learning. Another assumption of our model lies in using a global representative agent framework. In the real world policies such as renewables subsidies or carbon taxation are pursued by individual national governments that trade with each other. This may lead to collaboration, carbon leakage or a tragedy of the commons. A model that explores this problem in the context of strategic interaction is thus worth studying. We hope the future holds research in these directions.

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Appendix A. Appendix

Appendix A.1. Description of different regimes

In this section of the Appendix we derive the differential equations governing each of the regimes described above.

In the situation only oil is used, the F.O.C.'s that have to hold are:

$$U'(q) - G(S) = \lambda + \mu$$

$$\dot{\lambda} = \rho\lambda + G'(S)q, \dot{\mu} = \rho\mu - D'(E)$$

From this it follows the regime can be represented by three differential equations and two boundary conditions:

$$\dot{q} = \tfrac{\rho(U'(q)-G(S))-D'(E)}{U''(q)} \tag{A.1}$$

$$\dot{S} = -q, \qquad S(0) = S_0 \tag{A.2}$$

$$\dot{E} = q, \qquad E(0) = E_0 \tag{A.3}$$

Together with the condition from the simultaneous-fuse regime $b+\psi D'(E(T_1))/\rho = G(S(T_1)) + D'(E(T_1))/\rho$. In the case with simultaneous use of oil and coal, all F.O.C.'s must hold with equality:

$$U'(q+x) - G(S) = \lambda + \mu,\ U'(q+x) - b\ = \psi\mu,$$

$$\dot{\lambda} = \rho\lambda + G'(S)q,\ \dot{\mu} = \rho\mu - D'(E)$$

Together these lead to:

$$\dot{q} + \dot{x} = \frac{\rho(U'(q+x) - G(S)) - D'(E)}{U''(q+x)} = \frac{\rho(U'(q+x) - b) - \psi D'(E)}{U''(q+x)}$$

combining these gives:

$$b + \frac{\psi D'(E)}{\rho} = G(S) + \frac{D'(E)}{\rho} \tag{A.4}$$

We can use this equation to find $q(t)$ as a function of $x(t)$ by taking the time derivative. This leads to:

$$q(t) = \frac{\psi(\psi-1)D''(E)}{\rho(-G'(S) - \frac{\psi-1}{\rho}D''(E))}x(t) \tag{A.5}$$

Together with the usual $\dot{S} = -q$ and $\dot{E} = q + \psi x$, and the continuity of energy use at T_1: $q(T_1^+) + x(T_1^+) = q(T_1^-)$. Lastly, we examine the coal-only regime, The F.O.C.'s that have to hold reduce to:

$$\dot{x} = \frac{\rho(U'(x) - b) - \psi D'(E)}{U''(x)},$$

together with $\dot{S} = -q$ and $\dot{E} = q + \psi x$. If at T_2 after oil-coal a coal-only phase exists, the energy continuity equation reads $q(T_2^-) + x(T_2^-) = x(T_2^+)$. For our calibration this only happens in the market outcome.

Appendix A.2. Numerical Methods

Appendix A.2.1. Shooting

We use a Runge-Kutta method to simulate the oil-only regime. As we do not have an initial value of $q(0)$ it is impossible to give starting values for a simulation. Instead, we 'shoot', using a guess for $q(0)$ given some value for T_1. We then check the size of:

$$d = (q(T_1^+) + x(T_1^+) - q(T_1^-))^2 \tag{A.6}$$

To check how well the boundary equation at T_1 is held. By adjusting our guess for $q(0)$, we can find the unique minimum where $d = 0$ through a minimization routine. This then gives us the correct solution for $q(0)$ and so the optimal path of q,S and E $\forall t < T_1$, for given T_1.

Appendix A.2.2. Algorithm First Best

The following algorithm was used to simultaneously solve for the switch times between the three regimes in the first-best solution:

1. Guess the switch times T_1 and $T_2 > T_1$
2. As described above, use the Runge-Kutta method described above find $q(t), S(t), E(t) \forall t < T_1$. Given T_1 we can use the shooting algorithm compute $q(0)$ and the entire paths $q(t)$, $S(t)$ and $E(t)$ for $t < T_1$.
3. At the break of the oil-only regime and the oil-coal regime, calculate $q(T_1^+), x(T_1^+)$ through $q(T_1^-) = q(T_1^+) + x(T_1^+)$ and equation A.5.

4. In the oil-coal regime, use the Runge-Kutta method to find $x(t), q(t), S(t), E(t) \forall T_1 < t < T_2$. A more detailed account of how this is done can be found in section Appendix A.2.3.
5. For the renewables-only regime, everything is clear as $z(t) = (U')^{-1}(c)$, and $E(T_2) = E(t) = \frac{\rho(c-b)}{\psi\kappa} \forall t > T_2$
6. Extract $x(T_2^-), q(T_2^-), S(T_2^-), Y(T_2^-)$ from the previous step, and see if $x(T_2^-) + q(T_2^-) = z(T_2^+) = c$ and $E(T_2) = \frac{\rho(c-b)}{\psi\kappa}$ holds. If not, go back to step 1 and adjust the guesses for T_1 and T_2 until they do using a minimization algorithm (in our case the Nelder Mead minimization algorithm Nelder and Mead (1965)).

Appendix A.2.3. Runge-Kutta in the oil-coal phase

The goal is to find $x(t), q(t), S(t), E(t) \forall T_1 < t < T_2$ For instance, the Runge-Kutta method uses:

$$\dot{q} + \dot{x} = \frac{\rho(U'(q+x) - G(S)) - D'(E)}{U''(q+x)}$$

$$q(t) = \frac{\psi(\psi-1)D''(E)}{\rho(-G'(S) - \frac{\psi-1}{\rho}D''(E))} x(t) \equiv \Theta(S)x(t)$$

$$\dot{S} = -q$$

$$\dot{E} = q + \psi x$$

$$q(T_2^+) = \frac{q(T_1^-)}{1 + \Theta(S(T_1))}$$

$$x(T_2^+) = \frac{q(T_1^-)}{1 + \frac{1}{\Theta(S(T_1))}}$$

And $E(T_2), S(T_2)$ given from the oil-only regime.

We can approximate the solution of these equations through:

$$(x+q)(t+h) \approx (x+q)(t) + h(\dot{x} + \dot{q})$$

$$E(t+h) \approx E(t) + h\dot{E}$$

$$S(t+h) \approx S(t) + h\dot{S}$$

$$q(t+h) = \frac{(x+q)(t+h)}{1 + \Theta(S(t))}$$

$$x(t+h) = \frac{(x+q)(t+h)}{1+\frac{1}{\Theta(S(t))}}$$

and repeating $N = (T_2 - T_1)/h$ times, where at each repeat $t+h$ is set as the new t.

Appendix A.2.4. Algorithm Market Economy

The algorithm for the Market Economy is a bit simpler. The regime switches, (see also proposition 3) we have only two regimes: oil is used for as long as it is the cheapest fuel type, and then a switch is made to coal. This means simulation is simpler, as after the switch to coal energy use is given by $x(t) = U'^{-1}(b)$. Furthermore, we know at the switch time T, $x(T) = q(T) = U'^{-1}(b)$. The first-order conditions in the oil-only regime are given by:

$$U'(q) - G(S) = \lambda$$

$$\dot{\lambda} = \rho\lambda + G'(S)q,$$

as damages are not considered in the market. Together with $S(0) = S_0$, the system is uniquely determined and we can solve using Runge-Kutta methods.

Appendix A.2.5. Algorithm Renewables

For the oil-only regime finding the optimal energy consumption is slightly trickier. We assumed in 1 that initially oil is cheaper than coal and subsidized renewables $G(S_0) < b$ and $G(S_0) < c - \eta(0)$. It can be shown that (see section Appendix A.4) that oil and coal are not used together. Likewise, there is no simultaneous use of coal and renewables. Therefore, there are two possible scenarios:

1. First oil is used, then coal, and then renwables.
2. First oil is used, and then renewables.

Oil is used until $G(S(t)) = b$ or $G(S(t)) = c - \eta(t)$. Let T_1 be the regime switch time (when oil is phased out). If coal is phased in after oil

$$q(T_1^-) = x(T_1^+) = U'^{-1}(b),\ S(T_1) = G^{-1}(b)$$

Otherwise if renewables are phased in after oil, then

$$q(T_1^-) = z(T_1^+) = U'^{-1}(c - \eta(T_1)),\ S(T_1) = G^{-1}(c - \eta(T_1))$$

In the first case, we know that the market will switch to renewables only if the price, $c - \eta$ is lower than the market price for coal, b. Therefore, the social planner has only one policy choice: either set the subsidy constant at level $c-b$, or don't set a subsidy at all. In the second case, the market will switch to renewables when $U'(q) = U'(z)$ or $c = G(S) + \lambda$, where $\lambda = 0$ (see Withagen and van der Ploeg (2011)), so $b = G(S)$. The social planner has again only one choice, set no subsidy at all, or set it at constant level $c - G(S)$.

Thus we have two final conditions for the oil-only regime, conditional on which regime comes after oil-only. Then we can use the Runge-Kutta method to find optimal level q_0 and simulate it forward. Given the regime change we can find the optimal level of energy consumption. The regime sequence is what the policy-maker can influence with the subsidy $\eta(t)$. Let T_1 be the optimal switch time between the oil regime and the coal regime if $\eta(t) = 0$. Then, if the policy maker wants to influence the behavior of the market, he can either:

- At some $\bar{T} > T_1$ set $\eta(t) = c - b + \epsilon, \epsilon \to 0\ \forall t > \bar{T}$ replacing the coal-only with the renewables-only regime $\forall t > \bar{T}$
- At some point $\bar{T} < T_1$ set $\eta(t) = c - G(S(\bar{T}) + \epsilon, \epsilon \to 0$ replacing the oil-only with the renewables-only regime $\forall t > \bar{T}$, and avoiding coal-use entirely.

We can then write down the expression of social welfare, depending on $\bar{T}$

$$W = \int_0^{\bar{T}} e^{-\rho t}[U(\hat{q}(t)) - G(S(t))\hat{q}(t) - D(E(t))]dt + \int_{\bar{T}}^{\infty} e^{-\rho t}[U(U'^{-1}(b)) - D(E(\bar{T}))]dt \quad \bar{T} < T$$

$$W = \int_0^{\bar{T}} e^{-\rho t}[U(\hat{q}(t) + \hat{x}(t)) - G(S(t))\hat{q}(t) - b\hat{x}(t) - D(E(t))]dt + \int_{\bar{T}}^{\infty} e^{-\rho t}[U(U'^{-1}(b)) - D(E(\bar{T}))]dt \quad \bar{T} > T$$

All that remains is to find the optimal $\bar{T}$. We calculate welfare level using the expressions above for a range of levels of $\bar{T}$ and find the $\bar{T}$ for which that welfare is maximal.

Appendix A.3. Proof no Green Paradox if renewables are after coal

Proof. Suppose the switch to renewables occurs only after the switch to coal from oil. Then, we know that at the switch time from oil to coal, T,

$$G(S(T)) = b$$

We know this because in our optimization problem the choice between $e^{-\rho t}[U(q) - G(S)q]$ and $e^{-\rho t}[U(x) - bx]$ is maximized. Because q and x are perfect substitutes, we know that the above condition holds at the switch time. Furthermore, we know that $S(0) = S_0$, and in between we have the differential equations governing the regime:

$$\dot{S} = -q$$

$$\dot{q} = \frac{\rho(U'(q) - G(S))}{U''(q)}$$

In conclusion, the system is defined by two coupled first order differential equations and two boundary conditions on the time interval $[0, T]$. Thus, it is not dependent on the time of switching to renewables after introducing coal.

It remains to be proved that there is a green paradox if renewables are introduced before coal is introduced. For this, we refer the reader to proposition 4 in van der Ploeg and Withagen(2010) (Is there really a Green Paradox?) □

Appendix A.4. Proof no Simultaneous use in the Market and Second Best

Proof. If oil and coal are used together, we have:

$$U'(x + q) - G(S) = \lambda \tag{A.7}$$

$$\dot{\lambda} = \rho\lambda - G'(S)q \tag{A.8}$$

$$U'(x + q) \leq b, x \geq 0, c.s. \tag{A.9}$$

From this it directly follows that $G(S) = b$, which is can never be the case for more than an instant of time as oil is depleted when in use, and b is constant. Likewise, the conditions for the possibility of a coal-renewables regime reduce to $b = c - \eta$, which will only occur if the government sets the subsidy exactly equal to $\eta = c - b$, which will never happen as they can gain a finite amount of welfare for only infinitesimal cost of increasing the subsidy the tiniest bit. □

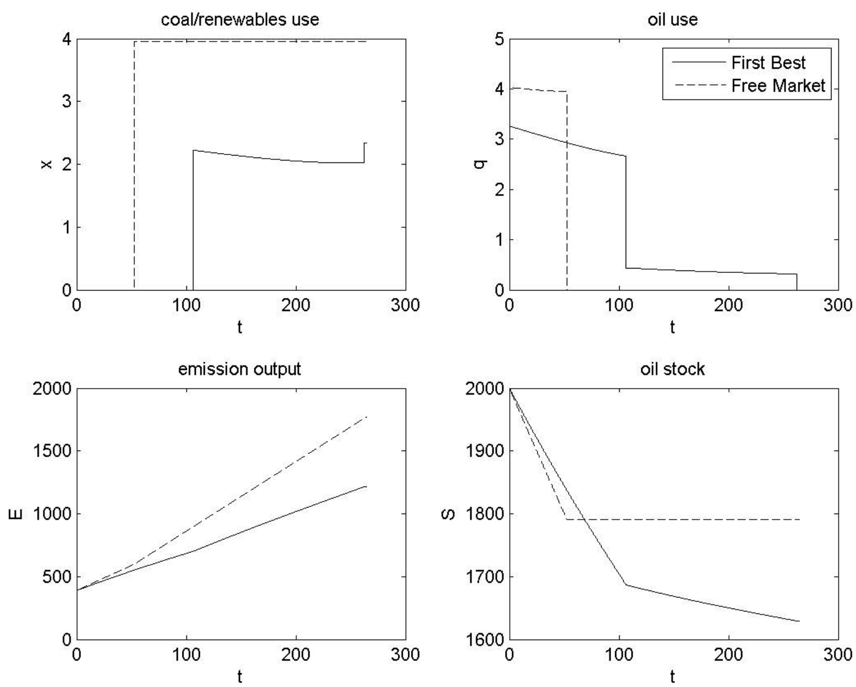

Figure A.1: coal/renewables use, oil use, emission output and oil stock as a function of time for both the social optimum and the market outcome

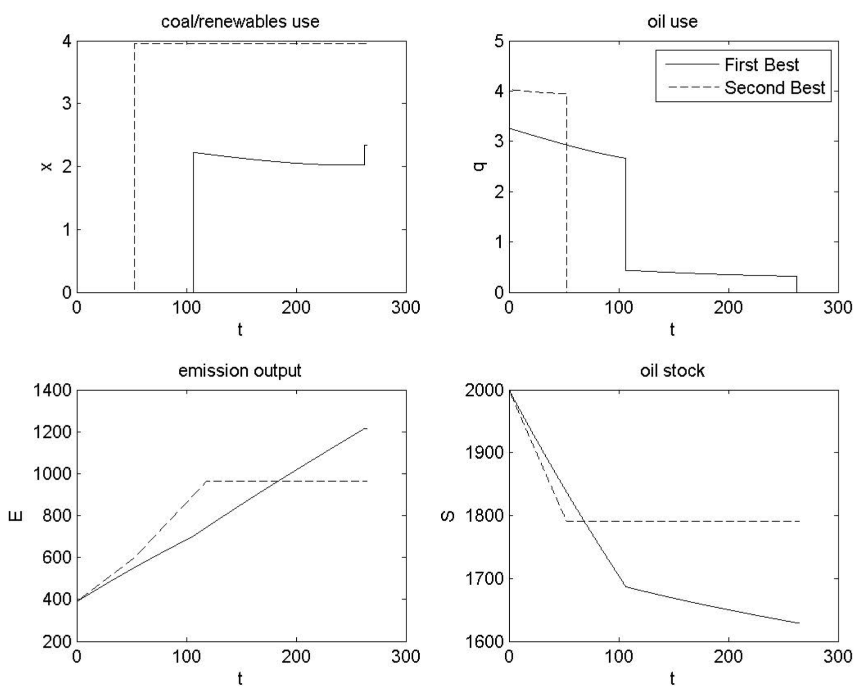

Figure A.2: coal/renewables use, oil use, emission output and oil stock as a function of time for the first-best and secondbest outcome

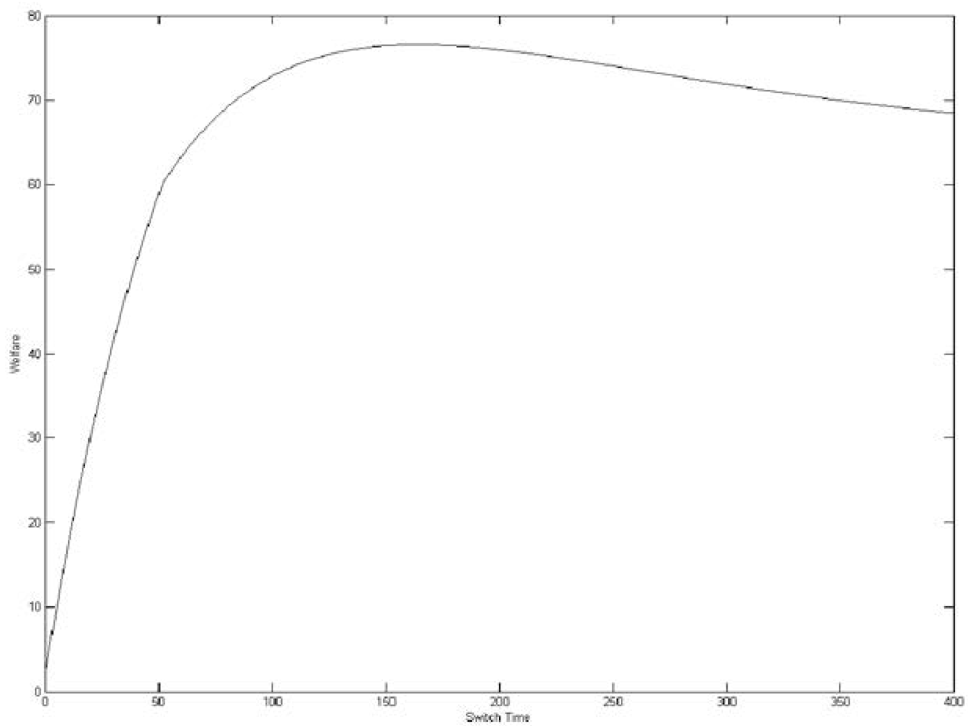

Figure A.3: Welfare as a function of the time subsidy is introduced.

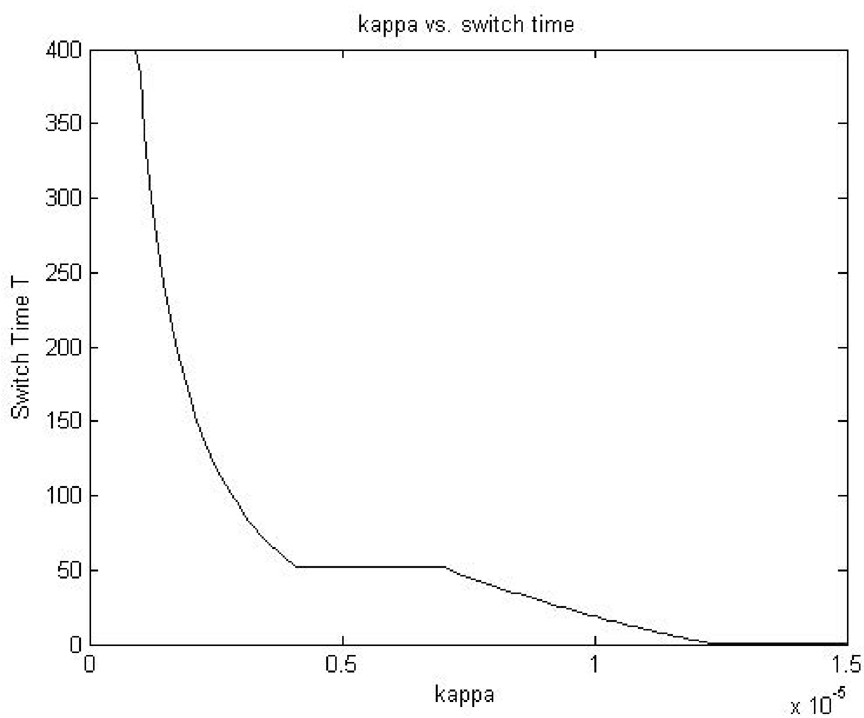

Figure A.4: How the optimal switch time to renewables depends on κ

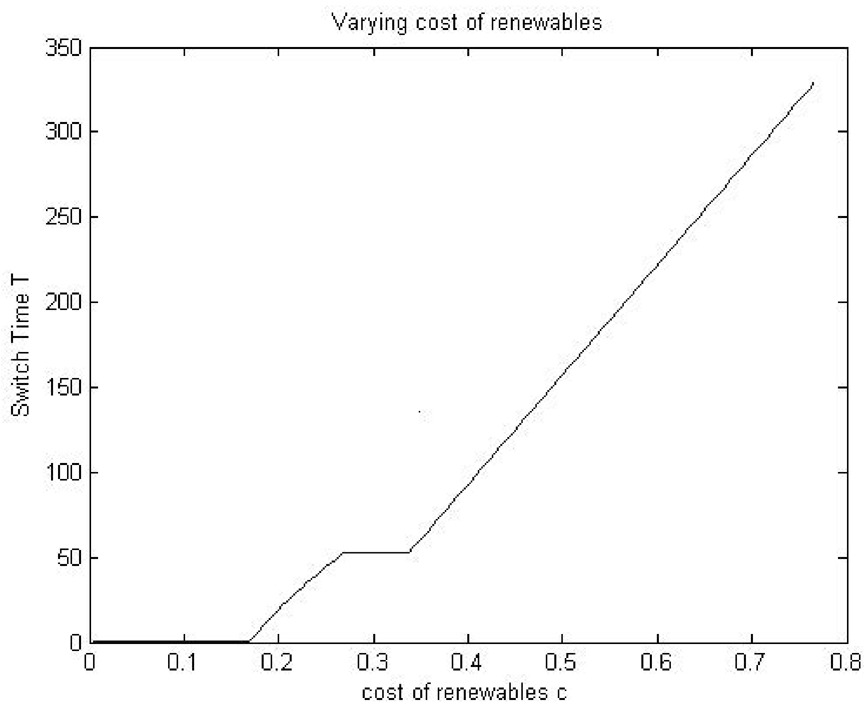

Figure A.5: How the optimal switch time to renewables depends on c

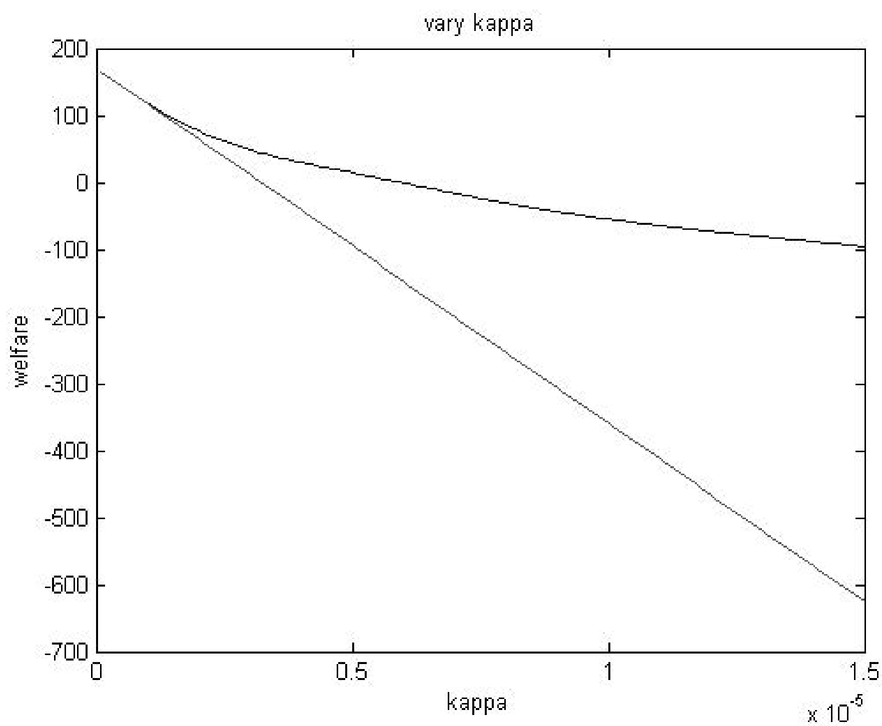

Figure A.6: How welfare in the optimal switch time to renewables depends on κ

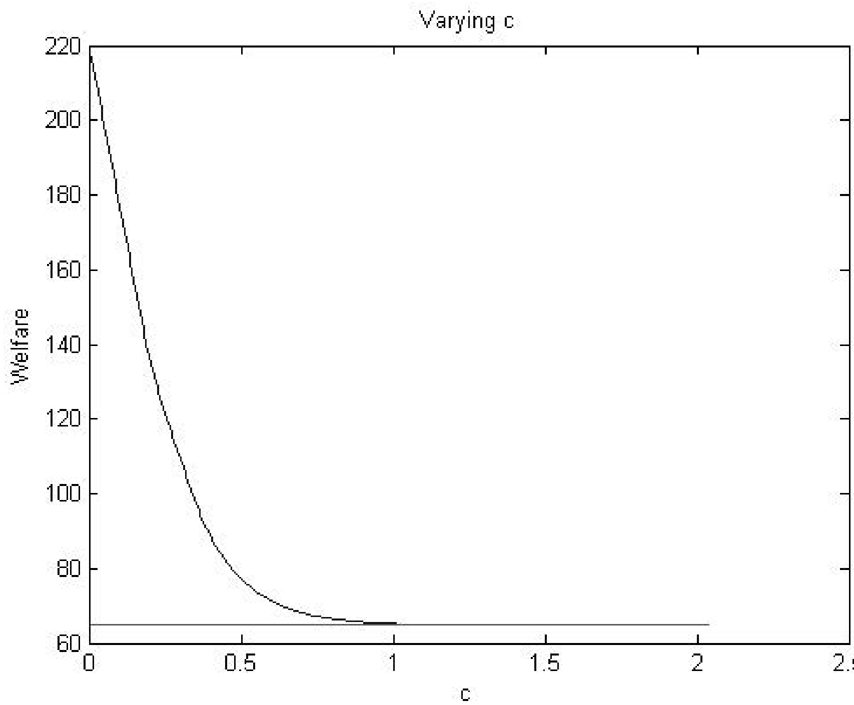

Figure A.7: How welfare in the optimal switch time to renewables depends on c